The PeRIPLO Propositional Interpolator

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Outline

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1. Formal Verification at USI, Lugano
2. Interpolation-based Model Checking
3. Flexible Interpolation Framework

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Formal Verification in Lugano, Switzerland

- Program Verification

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    - Model checking software (FunFrog, EvolCheck, LoopFrog), ANSI-C programs
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- Abstractions
  - Interpolation-based Bounded Model Checking
    - Function summarization [ATVA'12]
    - Upgrade checking, Incremental verification [FMCAD'13], [TACAS'13]
    - Recursion depth detection [STTT'15]
    - Verification-aided regression testing [ISSTA'13]

- Abstractions
  - Leveraging Interpolant strength [CAV'12]
  - Loop Summarization [ATVA'08], [ASE'09]
    - Program Termination [CAV'10], [TACAS'11]

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  - Synergy of Abstractions [STTT'10]

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Formal Verification in Lugano, Switzerland

- An SMT-based verification framework for software systems handling arrays [FMSD'15]
  - A quantifier-free interpolation procedure extending Lazy Abstraction [McMillan'06] to a quantified level [LPAR'12]
  - Identification of a class of relations over arrays admitting definable first-order acceleration [TACAS'13]
  - Booster: An Acceleration-Based Verification Framework for Array Programs [ATVA'14]

- Boolean and Theory Reasoning (SAT/SMT)

Background

Formal Verification in Lugano, Switzerland

- Boolean and Theory Reasoning (SAT/SMT)
  - Proof reduction and proof manipulation for interpolation [FMSD'15]
  - Proof Sensitive Interpolation [VSTTE'15]
  - Search-Space Partitioning for Parallelizing SMT Solvers [SAT'15]
  - Procedure for bit-vector extraction and concatenation [ICCAD'09]
  - Generation of explanations in theory propagation [MEMOCODE'10]

- Boolean and Theory Reasoning (SAT/SMT)
    - Solver, *OpenSMT*, combines MiniSAT2 SAT-Solver with state-of-the-art decision procedures for QF EUF, LRA, BV, RDL, IDL
        - *Extensible*: the SAT-to-theory interface facilites design and plug-in of new decision procedures
        - *Incremental*: suitable for incremental verification
        - *Open-source*: available under MIT license
        - *Parallelized*: efficient search space partitioning
        - *Efficient*: competitive open-source SMT Solver for QF UF, IDL, RDL, LRA according to SMT-Comp.

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  - Problem encoding into logic (SAT, SMT)
  - Problem solving by means of resolution based engines (SAT solvers, SMT solvers)

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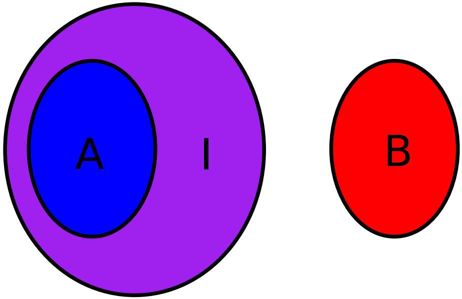

- Problems
  - Size affects efficiency
  - Interpolants different in their logical strength are needed
  - Collection of individual algorithms, no possibilities for adjustments wrt the model checking tasks

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Interpolation-based Model Checking

Motivation

- PeRIPLO is a multi-purpose interpolation framework

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    - proof manipulation for interpolation

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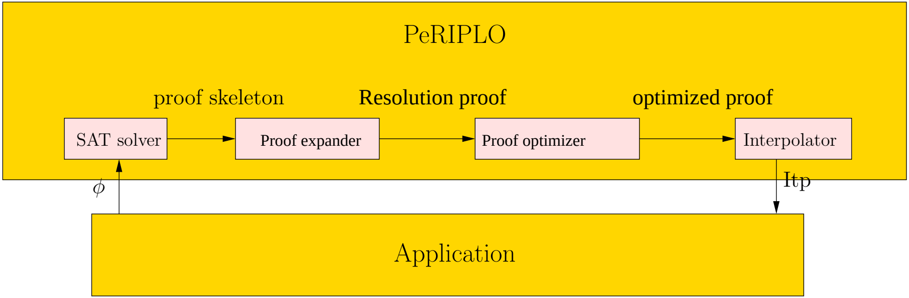
  - Proof senstive Interpolation

The Bird's Eye View to PeRIPLO



- Given an unsatisfiable propositional formula ϕ PeRIPLO constructs an interpolant in circuit form by

  1. Solving ϕ and extracting a compact proof skeleton from the SAT solver
  2. Expanding the proof skeleton to a resolution proof
  3. Optimizing the resolution proof to a smaller proof
  4. Constructing an interpolant from the optimized proof

PeRIPLO Features

- Basic interpolation:
  - $A \wedge B \rightarrow \bot$
  - $A \rightarrow I$ and $I \wedge B \rightarrow \bot$
  - $var(I) \subseteq var(A) \cap var(B)$

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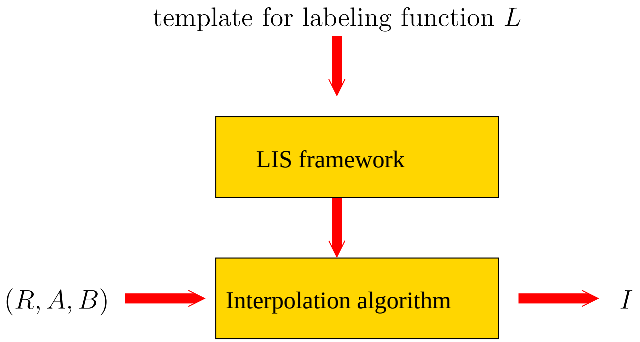
- Variations:
  - Path, Tree and DAG interpolation
- Proof Optimization [FMSD15, ICCAD10]:
  - removing resolution steps which reintroduce already resolved pivot variables
  - postponing unit resolution steps until the end of the resolution proof
  - using different local rewriting rules which preserve the validity of the proof

PeRIPLO Interface

Features

- Multifaceted interface
  - A clear API for C++ for tuning the interpolation algorithm, inserting a formula to PeRIPLO, and fetching the interpolant from PeRIPLO
  - Reading and writing smtlib2
  - Reading and writing the Aiger format

- Labeled Interpolation System (LIS) framework [D'Silva et al. 2010]
  - construction of interpolation algorithms from labeling functions
  - generalization of various interpolation algorithms (i.e., M_s [McMillan03], P [Pudlak97], M_w [D'Silva10])



- Given a resolution proof R, (v, C) denotes that the variable v occurs in a clause C of R
- The labeling function L assigns a label from the set $\{a, b, ab\}$ to each occurrence (v, C) in R
- Given a propositional formula $A \wedge B$, a variable is either *local*, if it occurs only in A or B, or *shared* if it occurs in both A and B

Interpolation in PeRIPLO

Labeling Functions [VSSTE15]

- The label $L(v, C) = b$ if v is local to A and $L(v, C) = a$ if v is local to B.
- The label can be chosen freely for occurrences of shared variables
  - *Tuning* the label for the shared occurrences results in *different* interpolation algorithms

PeRIPLO and Different Interpolation Algorithms

- Labeling all shared variable occurrences as
  - *a* results in the weakest interpolant M_w available in LIS
  - *b* results in a strong interpolant M_s available in LIS
  - *ab* results in an interpolant P that is somewhere in the middle
- The aforementioned approaches are fixed schemas with no possibility for adopt to the task

PeRIPLO and Proof-Sensitive Interpolation

- PeRIPLO offers certain labeling functions that specifically address the interpolant size:
  - Labeling all occurrences in A as a and B as b results in an interpolant with minimum number of distinct variables
  - By analyzing the number of occurrences in the A and B part of the proof R it is possible to construct interpolants that have a small number of connectives

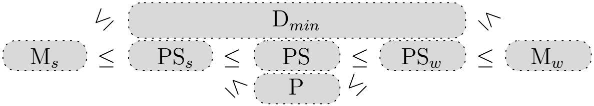
The Proof-Sensitive (PS) Labeling Functions

- PeRIPLO implements the proof-sensitive labeling functions specifically targeted for constructing small interpolants
- let $f_A(p) = |\{(p, C) \mid C \in A\}|$ be the number of times the variable p occurs in A, and $f_B(p) = |\{(p, C) \mid C \in B\}|$ the number the variable p occurs in B.
- The proof-sensitive labeling function L_{PS} is defined as

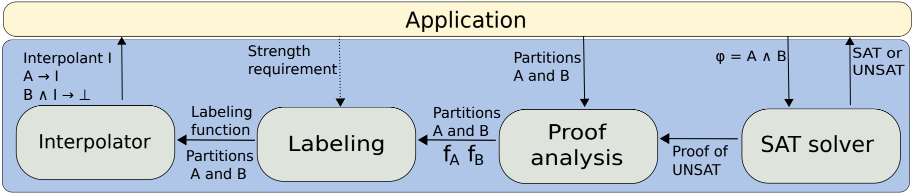
$$L_{PS}(p, C) = \begin{cases} a & \text{if } f_A(p) \geq f_B(p) \\ b & \text{if } f_A(p) < f_B(p). \end{cases}$$

The Proof-Sensitive Labeling Functions

- PeRIPLO also provides weak and strong versions of Proof-Sensitive Approach
- Hierarchy of Interpolation Algorithms provided by PeRIPLO



PeRIPLO API



- The API of PeRIPLO provides the application with a full control over the interpolant generation
- Many of the more routine tasks are implemented efficiently inside PeRIPLO so that the user does not need to take care of such details
- The system makes it comfortable to construct and experiment with new labeling functions

Reduction Approach Evaluation [ICCAD'10]

Experimental results over SMT: QF_UF, QF_IDL, QF_LRA, QF_RDL

	#	Avg_{nodes}	Avg_{edges}	Avg_{core}	$T(s)$	Max_{nodes}	Max_{edges}	Max_{core}
RP	1370	6.7%	7.5%	1.3%	1.7	65.1%	68.9%	39.1%
Ratio								
0.01	1366	8.9%	10.7%	1.4%	3.4	66.3%	70.2%	45.7%
0.025	1366	9.8%	11.9%	1.5%	3.6	77.2%	79.9%	45.7%
0.05	1366	10.7%	13.0%	1.6%	4.1	78.5%	81.2%	45.7%
0.075	1366	11.4%	13.8%	1.7%	4.5	78.5%	81.2%	45.7%
0.1	1364	11.8%	14.4%	1.7%	5.0	78.8%	83.6%	45.7%
0.25	1359	13.6%	16.6%	1.9%	7.6	79.6%	84.4%	45.7%
0.5	1348	15.0%	18.4%	2.0%	11.5	79.1%	85.2%	45.7%
0.75	1341	16.0%	19.5%	2.1%	15.1	79.9%	86.1%	45.7%
1	1337	16.7%	20.4%	2.2%	18.8	79.9%	86.1%	45.7%

- *Ratio* — time threshold as fraction of solving time
- # — number of benchmarks solved
- Avg_{nodes}, Avg_{edges}, Avg_{core} — average reduction in proof size
- $T(s)$ — average transformation time in seconds
- Max_{nodes}, Max_{edges}, Max_{core} — max reduction in proof size

Applications - FunFrog [FMCAD12] and eVolCheck [TACAS13]

- Bounded Model Checkers
- Interpolants used as Function Summaries
- FunFrog - C Incremental Checker
  - Stronger interpolants suit better [CAV12]
  - http://verify.inf.usi.ch/funfrog
- eVolCheck - C Upgrade Checker
  - Weaker interpolants suit better [CAV12]
  - http://verify.inf.usi.ch/evolcheck

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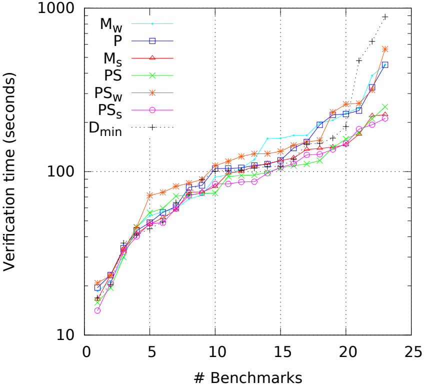

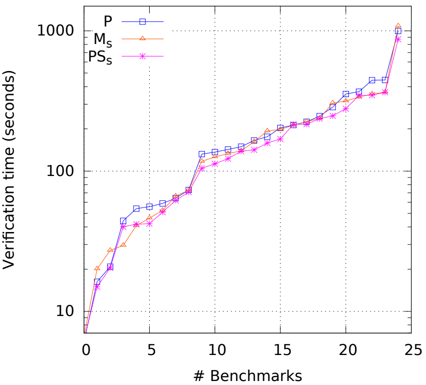

PS and PS_s consistently lead to better verification time

Conclusions

- PeRIPLO is an interpolation tool for propositional logic
  - Generic and flexible framework for producing interpolants on demand
  - Provides an API, an smtlib2, and an AIGER interface for communicating with other tools
  - Particular emphasis on constructing small interpolants while maintaining guarantees of interpolant strength
  - For more information see http://verify.inf.usi.ch/periplo!

Future work

- Extending the interpolation to first-oder logics in SMT

Thank you for your attention!